Investigating the Quantum Correlation Between Superconductivity, Resistance, and Energy Gap Phenomena

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ABSTRACT

When an external magnetic field applied on superconductor, the superconducting state is destroyed after the strength of magnetic field exceeds a certain critical value. This phenomenon can be explained on the basis of quantum resistance model. According to this model the critical magnetic field corresponds to the existence of an energy gap. The relation of this energy gap to the critical temperature resembles the conventional one. The existence of this gap can be explained on the basis of and Hubbard model.

Keywords*: Energy gap, superconductivity, resistance, plasma Equation quantum.*

I. INTRODUCTION

The phenomenon of superconductivity, in which the electrical resistance of certain materials completely vanishes at low temperatures, is one of the most interesting and sophisticated in condensed matter physics. Also The phenomenon of superconductivity has always been very exciting, both for its fundamental scientific interest and because of its many applications. These show that the energy gap exists in the superconducting [1].

The quantum (zero temperature) transition from a superconducting to a nonsuperconducting ground state is quantum phase transitions. This transition is induced by changing external parameters at zero temperature T [2,3]. There are two types of superconductors, I and II, characterized by the behavior in an applied magnetic field [4,5]. Type I superconductors depends on Critical Temperature T_c and Critical Magnetic Field B_c.

In the presence of an applied magnetic field B, the value of T_c decreases with increasing magnetic field for several type I superconductors.

When the magnetic field exceeds the critical field, B_c , the superconducting state is destroyed and the material behaves as a normal conductor with finite resistance. Type II also depends on two critical magnetic fields, designated B_{c1} and B_{c2}. When the external magnetic field is less than the lower critical field B_{c1}, the material is entirely superconducting and there is no flux penetration, just as with type I superconductors. When the external field exceeds the upper critical field B_{c2} , the flux penetrates completely and the superconducting state is destroyed, just as for type I materials. For fields lying between B_{c1} and B_{c2}, however, the material is in a mixed state, referred to as the vortex state [4].

Plasma equations describe ionized fluids subjected to electric and magnetic potentials for particles having thermal energy [6]. These equations are more generalized than Newton's equations for single particle[7]. Because it accounts for particles having thermal energy and moving in bulk matter [9].

Thus plasma equation is suitable for describing behavior of bulk matter
[9]. Thus it can be used to develop quantum equation for particles moving inside a certain medium [10]. Such equation can reduce quantum equation from large degrees of free dimension to 3dimensionson space only. Such equation was first developed by M.Dirar and Rasha. A [11]. This equation is used to explain some Schrödinger behavior, unfortunately this approach is complex mathematically. Thus there is a need for a simple model that can explain some Schrödinger phenomena. Section (2) is devoted for quantum equation derived from energy equation found from plasma equation. The solution and equation expression for resistance is exhibited in section (3). Section (4) is concerned with finding critical temperature, and superconductivity resistance and energy gap. Discussion and conclusion are in section (5) and (6) respectively.

II. COMPLEX QUANTUM RESISTANCE MODEL

Plasma equation describes ionized particles in a gaseous or liquid form. This equation can thus describes the electron motion easily. This is since the electrons be behaves as ionized particles in side matter. For pressure exerted by the gas plasma equation becomes:

$$mn\frac{dv}{dt} = -\nabla P + F \quad (1)$$

But for pressure exerted by the medium on the electron gas, the equation become:

$$mn\frac{dv}{dt} = \nabla P + F = \nabla P - \nabla V \quad (2)$$

In one dimensions, the equation becomes:

$$mn\frac{\mathrm{dv}}{\mathrm{dx}}\frac{\mathrm{dx}}{\mathrm{dt}} = \frac{d\,(\mathrm{nkT})}{\mathrm{dx}} - \frac{\mathrm{dnv}}{\mathrm{dx}}$$

$$mn\frac{vdv}{dx} = \frac{d}{dx}[\mathrm{nkT} - \mathrm{nv}]$$

where V is the potential for one particle

$$mn\frac{d1/2\,v^2}{dx} = \frac{d}{dx}[\mathrm{nkT} - \mathrm{nV}]$$

Thus in integrating both sides by assuming n to be constant, or in- dependent of K, yields:

$$\frac{n}{2}mv^2 = nkT - nV + c$$

$$\frac{1}{2}mv^2 + V - K\,T = \frac{c}{n} = constant = E$$

This constant of motion stands for energy, thus:

$$E = \frac{P^2}{2m} + V - kT \quad (3)$$

Multiplying by ψ, yields:

$$E\psi = \frac{P^2}{2m}\psi + V\psi - kT\psi \quad (4)$$

According to the wave nature of particles:

$$\psi = \mathrm{A}e^{\frac{i}{\hbar}(px - Et)}$$

$$i\hbar\frac{\partial\psi}{\partial t} = E\psi$$

$$-\hbar^2\nabla^2\psi = P^2\psi \quad (5)$$

$$i\hbar\frac{\partial\psi}{\partial t} = \frac{-\hbar^2}{2m}\nabla^2\psi + V\psi - kT\psi \quad (6)$$

The time in dependent equation becomes:

$$\frac{-\hbar^2}{2m}\nabla^2\psi + V\psi - kT\psi = E\psi \quad (7)$$

Consider the case when these electrons wave subjected to constant crystal field v_0.This assumption is quite natural as far as particles are distributed homogenously. Thus, equation (7) becomes:

$$\frac{-\hbar^2}{2m}\nabla^2\psi + V_0\psi - kT\psi = E\psi \quad (8)$$

One can suggest the solution to be:

$$\psi = Ae^{ikx} \quad (9)$$

A direct substitution yields:

$$(\frac{\hbar^2}{2m}k^2 + V_0 - kT)\psi = E\psi$$

Therefore:

$$K = \frac{\sqrt{2m(E+kT-V_0)}}{\hbar} \quad (10)$$

This wave number K is related to the momentum according to the relation :

$$P = mv = \hbar K = \sqrt{2m(E+kT-V_0)} \quad (11)$$

This relation can be used to find the quantum resistance R of a certain material . According to classical laws :

$$R = \frac{V}{I} \quad (12)$$

For electrons accelerated by the potential . the wave done is related to the potential V and kinetic energy K according to the relation :

$$w = V = \frac{1}{2}mv^2 \quad (13)$$

But since the current I is gives by .

$$I = nevA \quad (14)$$

$$R = \frac{mv^2}{2\,nev\,A} = \frac{mv}{2\,ne\,A} = \frac{P}{2\,ne\,A} \quad (15)$$

From (12) and (13) :

$$R = \frac{\sqrt{2m(E+kT-V_0)}}{2neA} \quad (16)$$

Splitting R to real part R_s and imaginary part $R_i R_i$:

$$R = R_s + R_i \quad (17)$$

According to equation (16) R becomes pure imaginary , when :

$$E = KT - V_0 < 0$$

$$KT < V_0 - E$$

$$T < \frac{(V_0 - E)}{K} \qquad (18)$$

Thus the critical temperature is given by :

$$T_c = \frac{V_0 - E}{K} \qquad (19)$$

Which requires:

$$V_0 > E$$

In this case (see equation (17)):

$$R = jR_i$$

$$R_s = 0 \qquad (20)$$

Thus the superconductivity résistance R_s Becomes zero beyond a certain critical temperature given by equation (17).Which requires binding energy to dominate.

III. ENERGY GAP AND PHOTON ABSORPTION

It is well known that in some superconductors, the materials behaves as an anti ferromagnetic this means that it is possible to consider electrons in the atoms as having spin up and spin down atoms with numbers N_u and N_d respectively, in the ground lower and excited states respectively, such that the magnetic flux density inside the medium is given by:

$$B_m = B_e(N_u - N_d) \qquad (21)$$

Where B_e is the magnetic flux density of one electron. If a photon beams was absorbed this will change B_m by the transition of electrons from ground state to the excited state. If the number of incident photons is N_p the new internal flux density is given by:

$$B_m = B_e(N_u - N_d + 2N_p) \qquad (22)$$

This change the potential of electrons to be V_m and split the energy levels to be

$$V_m = m_L \left(\frac{e\hbar}{2m}\right) B_m \qquad (23)$$

Thus the energy gap is given by:

$$E_g = \frac{e\hbar}{2m} B_m \qquad (24)$$

Here one assumes that any electron is affected by the magnetic field of the spinning electron gas. When electrons are affected by internal magnetic field the resistance in equation (16) and by the definition of T_c in equation (19) is given by:

$$R = \frac{\sqrt{2mK(T + T_m - T_c)}}{2neA} = R_s + jR_i \qquad (25)$$

Where

$$KT_m = V_m \quad (26)$$

The superconductivity is destroyed when,

$$T_m \geq T_c \quad (27)$$

Thus,

$$V_m \geq KT_c \quad (28)$$

Since V_m is proportional to B_m according to equation (24) the energy gap corresponds to the minimum voltage that destroy superconductivity. Thus,

$$E_g = c_m V_{mg} \quad (29)$$

Thus,

$$E_g = c_m V_{mg} \quad (30)$$

But according to equation (28) the minimum magnetic energy that can destroy superconductivity is,

$$V_{mg} = K T_c \quad (31)$$

Thus equation (30) indicates that the energy gap takes the form,

$$E_g = c_m K T_c \quad (32)$$

It is very interesting to note that this expression for E_g conforms with the well known ordinary relation. In this model the photon plays a double role. When it is incident and absorbed by the superconductivity it increases the internal field by causing more electrons with spin down to be in an excited state. This increase in the internal field B_m causes splitting of energy levels by the amount,

$$\Delta E = g m_s \mu_B H_m \quad (33)$$

IV. DISCUSSION

According to BCS theory cooper pairs are bound together by attractive force. The minimum energy which destroy the pairs and superconducting is called energy gap. It is clear that energy gap in equation (29) is related to energy splitting, in addition to the fact that the increase of magnetic field and magnetic potential above energy gap destroy conductivity. This means that according to cooper version energy gap represents an energy gap. It is very striking to note that the relation (32) for energy gap which relates it to the critical temperature a green with the conventional one. The effect of external magnetic field on superconducting can be also explained on the basis of Hubbard model by assuming that the external field increases Fermi energy. This increase can be explained on the basis of the relation between Fermi energy and free carriers concentration $E_F \propto n$ it is known that magnetic energy increases electrons energy. This can enable more electrons to be free by entering conduction band (CB) from the valence band (VB). According to hopping and mott insulator model, the Fermi level can be assumed in the lower continuous band. Thus no gap exists between (CB) and (VB) which are separated by Fermi level. Upon increasing external B, the free electrons and holes increases, thus E_F move upward till it creases the energy gap between the lower band and upper band. In this case the lower band which becomes (VB) and the upper band which becomes (CB). Thus an energy gap is produced between (VB) and (CB), which prevents electrons from hopping easily. Thus the material becomes an insulator

V. CONCLUSIONS

The quantum resistance model based on real and complex part can explain why the superconducting is destroyed when the external magnetic exceeds a critical value.

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