Vibration-Based Modal Analysis for Failure Detection in Cantilever Beams

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ABSTRACT

Modal analysis is performed to determine the vibration characteristics i.e. natural frequencies and mode shapes of structures. The natural frequencies and mode shapes are important parameters in the design for dynamic loading conditions. The knowledge of modal parameters is necessary to understand structural dynamics of structures. Modal parameters are global properties of a structure and any changes in these parameters can be used to detect and locate structural faults. The structural fault affects mass, damping and stiffness properties of structures. Modal parameters of structures could be obtained either by experimental modal analysis or from finite element analysis. In this research work, modal analysis of a mild steel cantilever beam has been carried out using finite element based software *ANSYS®* and the results of computational analysis are validated analytical. The calculated percentage difference of natural frequencies between computational analysis results and analytical results lies within the range of 1.5%. The stress concentration regions found on cantilever beam corroborated well with the failed zone.

KEYWORDS: *Mode shapes, Resonant frequency, Cantilever beam, ANSYS® Software.*

I. INTRODUCTION

The design of structures vibration applications demands an understanding of modal parameters. There are two ways to get modal parameters of structures. The modal parameters could be obtained either by finite element analysis or from experimental modal analysis. Today computational power is much larger, more reliable, and relatively cheap and as most technological related setups have access to computers, the popularity of using numerical methods is an ever increasing phenomenon. Especially finite element methods are being used at large extent for structural analysis. It is considered to be one of the best methods for solving a wide variety of practical problems efficiently. Finite element method has now become a very important tool of engineering analysis. Its versatility is reflected in its popularity among engineers and designers belonging to nearly all the engineering disciplines. The finite element method has become popular due to its relative simplicity of approach and accuracy of results. In the modern technological environment the conventional methodology of design cannot compete with the modern trends of Computer Aided Engineering (CAE) techniques (*Khawaja, 2007 [1]*). Various researchers have analyzed vibration and stress analyses problems using finite elements methods (*Ramamurti et al., 1998 [2]; Khan et al., 2006[3]; Krishnakanth et al., 2013 [4]*).

Finite element analysis (FEA) is a computerized method for predicting how a product reacts to real-world forces, vibration, heat, fluid flow and other physical effects. The methods has been extensively used in the field of structural mechanics; it has also been successfully applied to solve several other types of engineering problem, such as heat conduction, fluid dynamics, seepage flow, and electric and magnetic fields. Various software such as *Catia, Ansys, Pro-E, Solidworks* etc. are used for performing finite element modeling and analysis of structures. Finite element analysis (FEA) is used to perform static, dynamic/modal, harmonic and fatigue analysis of structures. *ANSYS®* has verified finite element methods by solving several problems and provided number of verification manuals related to static, modal, harmonic and fatigue analyses (*Zienwick et al., 1994 [5]). Yinming et al., 2004[6]* created and analyzed a CAD model of a cantilever. They have compared controlled and uncontrolled impulse responses at the free end of the beam in time domain and frequency domain. They have found that this proposed procedure can be used for solving complex structures problems. *Khan et al., 2013 [7]* analyzed a double cracked cantilever beam through finite element analysis. In this work, the modal parameters of a mild steel cantilever been has been obtained through dynamic analysis of beam using ANSYS® software. The computational result has been verified analytically.

II. COMPUTATIONAL ANALYSIS OF A MILD STEEL CANTILEVER BEAM

Computational analysis of structures has performed to evaluate structural and vibration characteristics of structures. A three dimensional CAD model of cantilever beam is created as shown in Fig. 1 through safe life design approach. Square surface mesh is made using auto meshing feature. The generated mesh modal of cantilever beam has 683 nodes and 80 elements as shown in Fig. 2. The boundary conditions are provided by making one end of cantilever beam fully built-in. The material properties and dimensions of mild steel cantilever beam are listed in Table 1and Table 2 respectively.

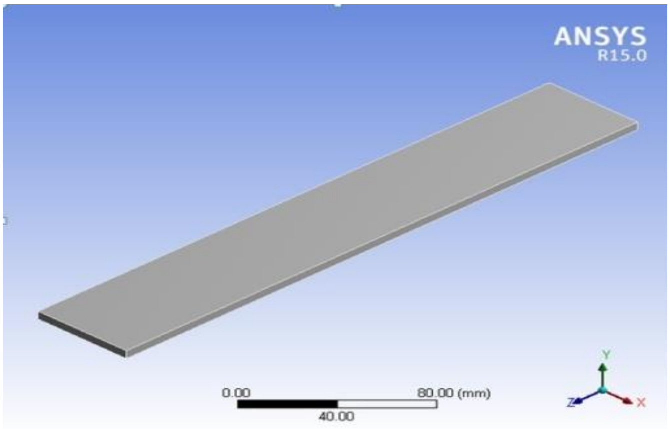


Fig. 1: CAD model of cantilever beam

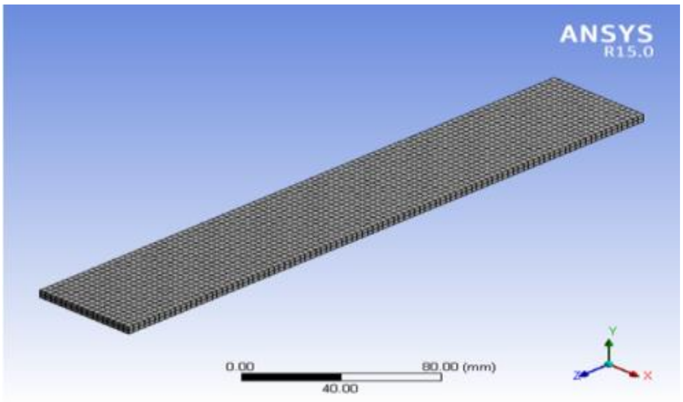


Fig. 2: Mesh model of cantilever beam

Table 1. Material properties of cantilever beam.

Material Properties	Cantilever beam
Young's Modulus, (N/m2)	2×10^{11}
Poisson's Ratio	0.3
Density (Kg/m^3)	7850
Bulk Modulus, (N/m^2)	1.1667×10^{11}
Shear Modulus, (N/m^2)	7.6923×10^{10}
Tensile Yield Strength, (N/m^2)	2.5×10^{8}
Tensile Ultimate Strength, (N/m^2)	4.6×10^{8}

Table 2. Dimensions of cantilever beam.

Material Properties	Cantilever beam
Length, L,(m)	0.29
Breadth, b,(m)	0.05
Depth, h,(m)	0.005

MODAL ANALYSIS OF CANTILEVER BEAM

Modal analysis is performed to determine the vibration characteristics i.e. natural frequencies and mode shapes of cantilever beam. The natural frequencies and mode shapes are important parameters in the design for dynamic loading conditions (*Shaikh et al., 2014 [8]; Lafta et al., 2014 [9])*. The first three resonant frequencies of beam are found at *49.20 Hz, 307.79 Hz,* and *861.46Hz* and their respective mode shapes are shown in Fig. 3(a),(b) and (c).

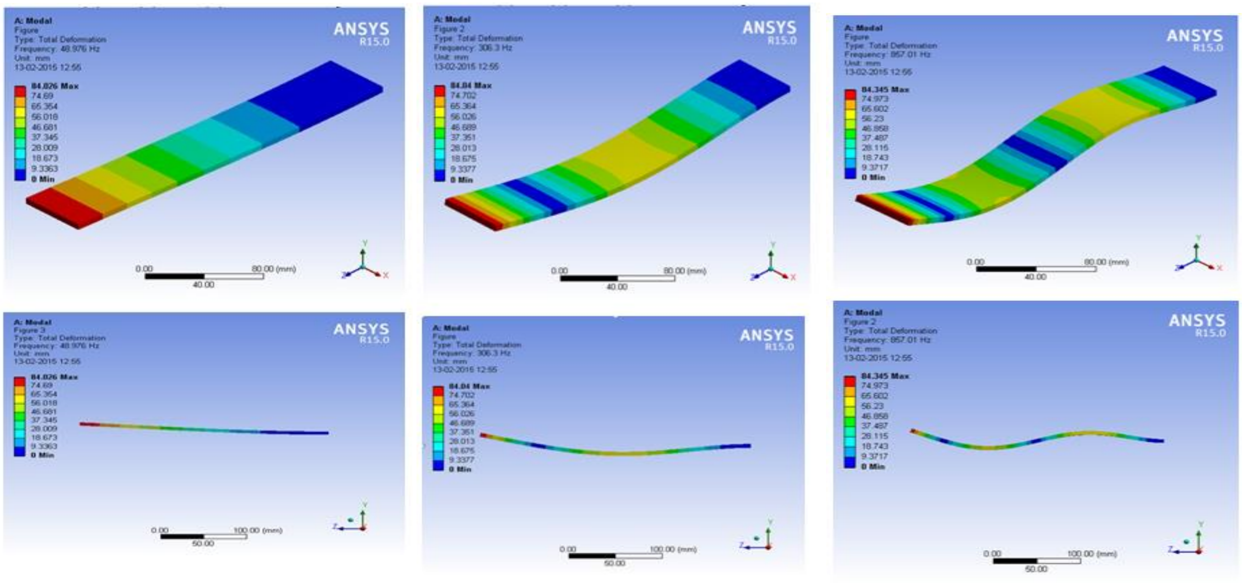


Fig. 3 (a): Mode 1 of cantilever beam (b) Mode 2 of cantilever beam (c) Mode 3 of cantilever beam

III. MODAL ANALYSIS OF CANTILEVER BEAM (ANALYTICAL)

For a cantilever beam Fig. (12), which is subjected to free vibration Fig. (13) and the system is considered as continuous system in which the beam mass is considered as distributed along with the stiffness of the shaft, the equation of motion can be written as

$$\frac{d^2}{dx^2}\left[EI(x)\frac{d^2y(x)}{dx^2}\right]=\omega^2 m(x) \tag{1}$$

Where, *E* is the modulus of rigidity of cantilever beam material, *I* is the moment of inertia of the beam cross-section, *Y*(*x*) is displacement in *y* direction at distance *x* from fixed end, *ω* is the natural frequency, *m* is the mass per unit length, *m = ρA(x)*, ρ is the material density, x is the distance measured from the fixed end.
Boundary conditions for cantilever beam are

$$\text{At } x=0,\ y(x)=0,\ \frac{dy(x)}{dx}=0 \text{ and At } x=l,\ \frac{d^2y(x)}{dx^2}=0,\ \frac{d^3y(x)}{dx^3}=0 \tag{2}$$

From equation of motion, we get

$$\frac{d^4y(x)}{dx^4}-\beta^4 y(x)=0,\quad \text{where}\ \beta^4=\frac{\omega^2 m}{EI} \tag{3}$$

The mode shapes of a cantilever beam is given as

$$f_n(x)=A_n\{(\sin\beta_nL-\sinh\beta_nL)(\sin\beta_nx-\sinh\beta_nx)+(\cos\beta_nL-\cosh\beta_nL)(\cos\beta_nx-\cosh\beta_nx)\} \tag{4}$$

Where n=0, 1, 2, 3 ...∞ and β_n L= nπ

Natural frequency of cantilever beam using equations (15) and (16) can be written as

$$\omega_n=\alpha_n^2\sqrt{\frac{EI}{mL^4}} \qquad \text{Where } \alpha_n = 1.875, 4.694, 7.85 \tag{5}$$

The first three resonant frequencies of cantilever beam having same length (L) 0.29m, breadth (b) 0.05m and depth (h) 0.005m are calculated using Eq. (5) are 49.69Hz, 311.40Hz, 872.01Hz. The analytical results are tabulated in Table 3.

Table 3: Analytical results for modal frequencies of cantilever beam

Natural frequency	Formula	Natural frequency (rad/sec)	Natural frequency (Hz)
1.	$\omega_1=(1.875)^2\times\sqrt{\frac{EI}{\rho AL^4}}$	$\omega_1=(1.875)^2\times\sqrt{\frac{2\times10^{11}\times0.005^2}{12\times7850\times0.29^4}}$ =312.053 rad/sec	ω_1 =49.69Hz

2.	$\omega_2 = (4.694)^2 \times \sqrt{\frac{EI}{\rho A L^4}}$	$\omega_2 = (4.694)^2 \times \sqrt{\frac{2\times10^{11}\times0.005^2}{12\times7850\times0.29^4}}$ = 1955.592 rad/sec	ω_2 = 311.40 Hz
3.	$\omega_3 = (7.855)^2 \times \sqrt{\frac{EI}{\rho A L^4}}$	$\omega_3 = (7.855)^2 \times \sqrt{\frac{2\times10^{11}\times0.005^2}{12\times7850\times0.29^4}}$ = 5476.222 rad/sec	ω_3 = 872.01 Hz.

IV. RESULTS AND DISCUSSION

The comparison of natural frequencies between computational modal analysis results and analytical results is given in Table 4. The computational modal analysis results of beam are corroborated well with the analytical results. The calculated percentage difference of natural frequencies between the analytical results and computational modal analysis results lies within the range of 1.5 %. The graphical representation of comparison is shown in Fig. 4.

Table 4. Comparison between computational and analytical modal analysis results.

Mode	Analytical Results Frequency [Hz]	Computational Results Frequency [Hz]	Difference [%] b/w Computational &Analytical Results
Mode 1	49.69	49.20	0.98
Mode 2	311.40	307.79	1.15
Mode 3	872.01	861.46	1.20

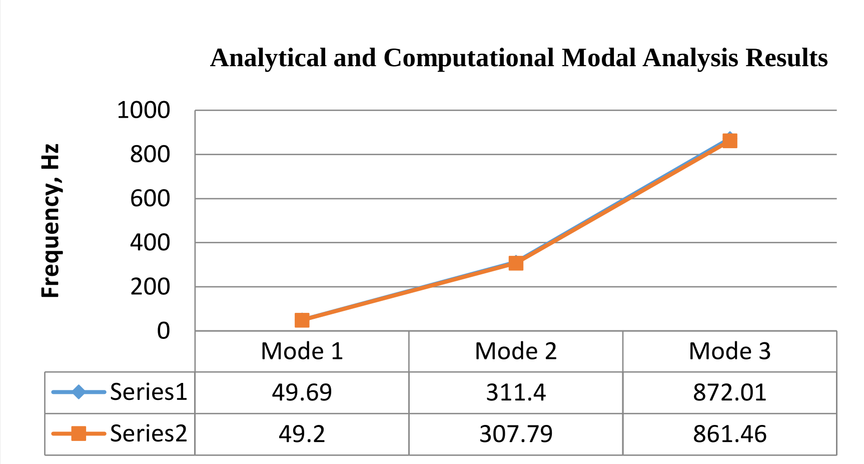
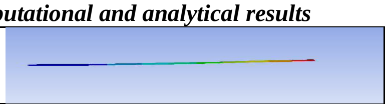
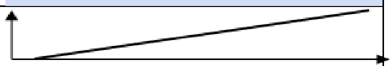
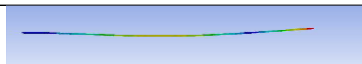
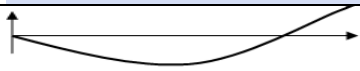
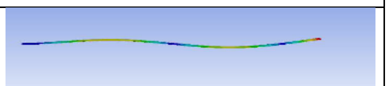
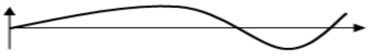


Fig. 4: Comparison of Analytical and Computational modal analysis results

The comparisons of mode shapes of computational and analytical modal analysis results are tabulated in Table 5.

Table 5. Comparison of mode shapes of computational and analytical results

1.	Computational	$\omega_1 = 49.20\text{Hz}$	
	Analytical	$\omega_1 = 49.69\text{Hz}$	
2.	Computational	$\omega_2 = 307.79\text{Hz}$	
	Analytical	$\omega_2 = 311.40\text{Hz}$	
3.	Computational	$\omega_4 = 861.46\text{Hz}$	
	Analytical	$\omega_4 = 872.01\text{Hz}$	

V. CONCLUSIONS

In this work, modal analysis of cantilever beam has been carried out both computationally and analytically. The following conclusions have been drawn:

1. Modal analysis is performed to determine the vibration characteristics i.e. natural frequencies and mode shapes of a mild steel cantilever beam using finite element based software *ANSYS®* and analytically.
2. The computational modal analysis results of beam are corroborated well with the analytical results.
3. The calculated percentage difference of natural frequencies between the analytical results and computational modal analysis results lies within the range of 1.5 %.
4. The knowledge of modal parameters is necessary to understand structural dynamics of structures.
5. The stress concentration regions found on cantilever beam corroborated well with the failed zone.

IV. REFERENCES

[1] Khawaja H.A., Khan A.M., “Modal and harmonic analysis of L.P. turbine of a small turbofan engine using finite element method”, FAILURE OF ENGINEERING MATERIALS & STRUCTURES, pp. 22-27, 2007.
[2] Ramamurti V., Subramani D. A., Sridhara K., “Free vibration of a turbocharger centrifugal compressor impeller”, MECHANISM AND MACHINE THEORY, Vol. 30, No. 4, pp. 619-628, 1995.
[3] Khan A.M., Ahmed M.N., Mustaq S., “Stress analysis of a major load bearing frame of medium weight aircraft using finite element method”, JOURNAL OF ENGINEERING & APPLIED SCIENCE, Vol. 25, No. 1, pp. 47-55, 2006.
[4] Krishnakanth P.V., Raju G.N., Prasad RDV., Saisrinu R., “Structural & thermal analysis of gas turbine blade by using F.E.M.” INTERNATIONAL JOURNAL OF SCIENTIFIC RESEARCH ENGINEERING & TECHNOLOGY, Vol. 2, No. 2, pp. 060-065, 2013.
[5] Zienwick O.C., Taylor R.L., “The finite element method”, fourth edition (1994); McGraw Hill.
[6] Yinming S., Hongxing H., and Hugo S., “The finite element analysis and experimental study of beams with active constrained layer damping treatments”, JOURNAL OF SOUND AND VIBRATIONS, Vol. 278, No. 1, pp. 343-363, 2004.
[7] Khan I.A., Parhi D.R.,“Finite Element Analysis of double cracked beam and its experimental validation”, PROCEDIA ENGINEERING, Vol. 53, pp. 703-708, 2013.
[8] Shaikh N., Rathod W.S., Ansari K., “Modal and harmonic analysis of turbocharger turbine using finite element method”, INTERNATIONAL JOURNAL OF APPLICATION OR INNOVATION IN ENGINEERING AND MANAGEMENT, Vol. 2, No. 7, pp. 6-9, 2013.
[9] Lafta H.S., Gupta A.V.S.S.K.S., “Finite element analysis of a gas turbine rotor blade”, INTERNATIONAL JOURNAL OF SCIENTIFIC ENGINEERING AND TECHNOLOGY RESEARCH Vo. 3, No. 20, pp. 4173-4180, 2014.
[10] Cook R.D., “Finite element method for stress analysis”, John Wiley and Sons (1995).