

STRONGLY CHROMATIC METRO DOMINATION OF P_n , C_n AND P_n^2

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ABSTRACT

A dominating set D of a graph $G(V, E)$ is called metro dominating set G if for every pair of vertices u, v , there exists a vertex w in D such that $d(u, w) \neq d(v, w)$. A metro dominating set D is called strongly chromatic metro dominating set if for every vertex $v \in D$ is from the same color class. The minimum cardinality strongly chromatic metro dominating set is called strongly chromatic metro domination number and is denoted by $SC\gamma_\beta$. In this paper we find strongly chromatic metro domination number of path, cycles and square of a path.

Keywords: metric dimension, metro domination, strongly chromatic metro domination, power graph.

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I. INTRODUCTION

Let $G(V, E)$ be a simple, non-trivial, undirected and non-null graphs. A graph G is k -colorable if there exists a k -coloring of G . One of the fastest growing areas within graph theory is the study of domination and related problem. A subset D of V is said to be a dominating set of G if every vertex in $V-D$ is adjacent to a vertex in D .

The minimum cardinality of a dominating set is called the domination number of G and is denoted by $\gamma(G)$. A subset D of V is said to be a dom-chromatic set if D is a dominating set and $\chi(\langle D \rangle) = \chi(G)$. The dom-chromatic number $\gamma_{ch}(G)$ of G is the minimum cardinality of a dom-chromatic set.

In 1976 F. Harary and R.A. Melter [1] introduced the notation of metric dimension. A vertex $x \in V(G)$ resolves a pair of vertices $u, w \in V(G)$ if $d(v, x) \neq d(w, x)$. A set of vertices $S \subseteq V(G)$ resolves G and S is a resolving set of G , if every pair of distinct vertices of G are resolved by same vertex in S . A resolving set S of G with minimum cardinality is a metric dimension of G denoted by $\beta(G)$.

A dominating set D of $V(G)$ having a property that for each pair of vertices u, v there exist a vertex w in D such that $d(u, w) \neq d(v, w)$ is called metro dominating set of G or simply MD-set. The minimum cardinality of a metro dominating set of G is called metro domination number of G and is denoted by $\gamma_\beta(G)$.

II. DEFINITIONS

2.1 Metric dimension:

The metric dimension of a graph G is the minimum cardinality of a subset S of vertices such that all other vertices are uniquely determined by their distances to the vertices in S . It is denoted by $\beta(G)$.

2.2 Domination:

Let $G(V, E)$ be a graph. A subset of vertices $D \subseteq V$ is called a dominating set (γ -set) if every vertex in $V-D$ adjacent to atleast one vertex in D . The minimum cardinality of a dominating set is called the domination number of the graph G and is denoted by $\gamma(G)$.

2.3 Locating domination:

A dominating set D is called a locating dominating set or simply LD-set if for each pair of vertices $u, v \in V-D$, $ND(u) \neq ND(v)$ where $ND(u) = N(u) \cap D$. The minimum cardinality of an LD-set of the graph G is called the locating domination number of G denoted by $\gamma_L(G)$.

2.4 Metro domination:

A dominating set D of $V(G)$ having the property that for each pair of vertices u, v there exists a vertex w in D such that $d(u, w) \neq d(v, w)$ is called metro dominating set of G or simply MD-set. The minimum cardinality of a metro dominating set of G is called metro domination number of G and is denoted by $\gamma_\beta(G)$.

2.5 Chromatic number:

The minimum number of colors required for a proper coloring of G is called chromatic number of G and is denoted by $\chi(G)$.

2.6 Chromatic domination:

A subset D of V is said to be a dom-chromatic set if D is a dominating set and $\chi(\langle D \rangle) = \chi(G)$. The dom-chromatic number $\gamma_{ch}(G)$ of G is the minimum cardinality of a dom-chromatic set.

III. SOME KNOWN RESULTS

In this section we mention some of the known result on metric dimension, domination, metro domination.

Theorem 3.1. (Harary and Melter [1]) The metric dimension of a non trivial complete graph of order n is $n-1$.

Theorem 3.2. (Khuller, Raghavachari, Rosenfeld [4]) The metric dimension of a graph G is 1 if and only if G is a path.

Theorem 3.3. (Harary and Melter [1]) The metric dimension of a complete bipartite graph $K_{m,n}$ is $m+n-2$.

Theorem 3.4.[5] The metro domination number of a graph G is $\left\lceil \frac{n}{5} \right\rceil$ if and only if G is a cycle.

Theorem 3.5.[5] Let G be a graph on n vertices. Then $\gamma_\beta(G) = n-1$ if and only if G is K_n or $K_{1,n-1}$ for $n \geq 1$.

Theorem 3.6. [5] For any integer n , $\gamma_\beta(P_n) = \left\lceil \frac{n}{3} \right\rceil$.

Remark 3.7. For any connected graph G , $\gamma_\beta(G) \geq \max\{\gamma(G), \beta(G)\}$.

Remark 3.8. For any integer $n > 3$, $\chi(C_n) = \begin{cases} 3 & \text{for } n \text{ odd} \\ 2 & \text{for } n \text{ even} \end{cases}$

Remark 3.9. For any integer $n > 1$, $\chi(P_n) = 2$.

Lemma 3.10. [9] Let $G = P_n^2$, $n > 3$. Then $\dim(G) = 2$.

Theorem 3.11.[7] For every $n \geq 1$, $\gamma_\beta(P_n^2) = \left\lceil \frac{n}{5} \right\rceil$.

Theorem 3.12. [2] For any integer $n \geq 3$, $\gamma_\beta(P_n^2) = \begin{cases} 2 & \text{if } 3 \leq n \leq 7 \\ 3 & \text{if } 8 \leq n \leq 10 \\ \left\lceil \frac{n}{5} \right\rceil & \text{if } n \geq 11 \end{cases}$

Remark 3.13. For any integer $n \geq 3$, $\chi(P_n^2) = 3$.

IV. MAIN RESULTS

Theorem 4.1. For any integer $n \geq 4$, $SC\gamma_\beta(P_n) = \left\lceil \frac{n-1}{2} \right\rceil$.

Proof: By theorem 3.2 $\beta(P_n) = 1$ and by remark 3.9 $\chi(P_n) = 2$, clearly we have $\left\lceil \frac{n}{2} \right\rceil$ vertices of one color class and remaining $\left\lfloor \frac{n}{2} \right\rfloor$ vertices of other color class. Hence we have choice of either $\left\lfloor \frac{n}{2} \right\rfloor$ or $\left\lceil \frac{n}{2} \right\rceil$ vertices for dominating set D

whose vertices are from the same color class. For even n , $\frac{n}{2}$ vertices of same color class dominates the remaining $\frac{n}{2}$ vertices. For odd n , $\frac{n-1}{2}$ vertices of same color class will dominates the remaining vertices and hence $SC\gamma_\beta(P_n) \geq \left\lfloor \frac{n-1}{2} \right\rfloor$ (1)

To prove the reverse inequality, we define a strongly chromatic dominating set $D = \{v_{2i} / 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor\}$ of cardinality $\left\lfloor \frac{n-1}{2} \right\rfloor$. We note that D acts as a dominating set also as a resolving set and each $v_i \in D$ is from the same color class and hence $SC\gamma_\beta(P_n) \leq \left\lfloor \frac{n-1}{2} \right\rfloor$ (2)

from (1) and (2)

$$SC\gamma_\beta(P_n) = \left\lfloor \frac{n-1}{2} \right\rfloor.$$

Theorem 4.2. For any integer $n \geq 5$, $SC\gamma_\beta(C_n) = \left\lfloor \frac{n-1}{2} \right\rfloor$.

Proof: By the result $\beta(C_n) = 2$ and by remark 3.8, $\chi(C_n) = \begin{cases} 3 & \text{for } n \text{ odd} \\ 2 & \text{for } n \text{ even} \end{cases}$, clearly we have $\frac{n}{2}$ vertices of one color class and remaining $\frac{n}{2}$ vertices of other color class for even n and $\left\lfloor \frac{n}{2} \right\rfloor$ vertices of one color class and other $\left\lfloor \frac{n}{2} \right\rfloor$ vertices of second color class and remaining one vertex of third color class for odd n . Hence we have choice of $\frac{n}{2}$ vertices for dominating set D such that each $v_i \in D$ are from the same color class. For even cycle, $\frac{n}{2}$ vertices of same color class dominate the remaining $\frac{n}{2}$ vertices. For odd cycle, $\frac{n-1}{2}$ vertices of same color class will dominate the remaining vertices and hence $SC\gamma_\beta(C_n) \geq \left\lfloor \frac{n-1}{2} \right\rfloor$ (1)

To prove the reverse inequality, we define a strongly chromatic dominating set $D = \{v_{2i-1} / 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor\}$ of cardinality $\left\lfloor \frac{n-1}{2} \right\rfloor$, which also acts as a resolving set and each $v_i \in D$ is from the same color class and hence $SC\gamma_\beta(C_n) \leq \left\lfloor \frac{n-1}{2} \right\rfloor$ (2)

from (1) and (2)

$$SC\gamma_\beta(C_n) = \left\lfloor \frac{n-1}{2} \right\rfloor.$$

Theorem 4.3. For any integer $n \leq 9$, $SC\gamma_\beta(P_n^2) = \left\lfloor \frac{n}{3} \right\rfloor$.

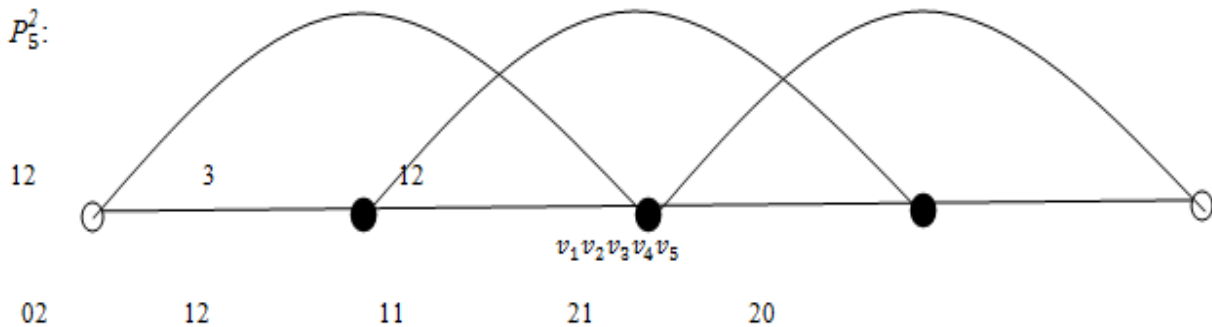
Proof: By lemma 3.10, $\dim(P_n^2) = 2$ for $n > 3$. Also by Theorem 3.11 $\gamma_\beta(P_n^2) = \left\lfloor \frac{n}{5} \right\rfloor$, $n \geq 11$ here each $\left\lfloor \frac{n}{5} \right\rfloor$ vertices of metro dominating set are not from the same color class. By remark 3.13, $\chi(P_n^2) = 3$, $n \geq 3$ if we label v_1 of P_n^2 by color 1 and v_2 by color 2 and v_3 by color 3 and continuing the coloring, we get $\left\lfloor \frac{n}{3} \right\rfloor$ vertices of color class 1, $\left\lfloor \frac{n+1}{3} \right\rfloor$ vertices of color class 2 and $\left\lfloor \frac{n}{3} \right\rfloor$ vertices of color class 3. Hence we have a choice of $\left\lfloor \frac{n}{3} \right\rfloor$ or $\left\lfloor \frac{n+1}{3} \right\rfloor$ or $\left\lfloor \frac{n}{3} \right\rfloor$ vertices for strongly chromatic metro dominating set minimum among these $\left\lfloor \frac{n}{3} \right\rfloor$ is minimum and hence $SC\gamma_\beta(P_n^2) \geq \left\lfloor \frac{n}{3} \right\rfloor$ (1)

To prove the reverse inequality, we defined the strongly chromatic metro dominating set as $D = \{v_{3i} / 1 \leq i \leq \left\lfloor \frac{n}{3} \right\rfloor\}$ of cardinality $\left\lfloor \frac{n}{3} \right\rfloor$. We note that D is a dominating set also acts as a resolving set and each $v_i \in D$ are all from the same color class and hence $SC\gamma_\beta(P_n^2) \leq \left\lfloor \frac{n}{3} \right\rfloor$ (2)

from (1) and (2)

$$SC\gamma_\beta(P_n^2) = \left\lceil \frac{n}{3} \right\rceil.$$

EXAMPLE:



$$D_1 = \{v_3\}$$

D_1 is a dominating set but not resolving set.

$$D_2 = \{v_1, v_5\}$$

D_2 is a dominating set also resolving set but both vertices are not from same color class. Hence it is not a strongly chromatic metro domination.

Hence P_5^2 is not a strongly chromatic metro domination.

REFERENCE

1. F. Harary and R.A. Melter, on the metric dimension of graphs, *Ars Combinatoria*, 2(1976) 191-195.
2. G.C. Basavaraju, M. Vishukumar and P. Raghunath, Metro domination of Square Path, *Annals of Pure and Applied Mathematics* vol.14, No. 3, 2017, 539-545.
3. S. Balamurugan, P. Aristotle, V. Swaminathan and G. Prabakaran, on graphs whose neighbourhood chromatic domination number is two, *International Journal of Pure and Applied Mathematics*, Volume 119, No. 15, 2018, 213-224.
4. S. Khuller, B. Raghavachari and A. Rosenfeld, Landmarks in graph, *Discrete Appl. Math.*, 70 (3) (1996) 217-229.
5. P. Raghunath and B. Sooryanarana, Metro domination number of a graph, *Twentieth annual conference of Ramanujan Mathematics Society*, July 25-30 (2005) University of Calicut.
6. S. Lakshminarayana and M. Vishukumar, on the k -Metro domination number of Paths, *Annals of Pure and Applied Mathematics*, Vol. 14, No. 3, 2017, 593-600.
7. M. Alishahi and S.H. Shalmaee, Domination number of Square of Cartesian product of Cycles, *Open Journal of Discrete Mathematics*, 5(2015) 88-94.
8. G.C. Basavaraju, M. Vishukumar and P. Raghunath, Metro domination of Square Cycle, *International Journal of Mathematics and its Applications*, Volume 5, Issue 4-E(2017), 641-645.
9. M. M. Al-Holi, O.A. Abughneim and H.A. Ezech, Metric dimension of Path related graphs, *Global Journal of Pure and Applied Mathematics*, 13 (2) (2017) 149-157.